REMOVING MIXTURE NOISE FROM MEDICAL IMAGES USING BLOCK MATCHING FILTERING AND LOW-RANK MATRIX COMPLETION

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ABSTRACT

Denoising as one of the most significant tools in medical imaging was studied widely in the literature. However, most existing medical image denoising algorithms have assumed the additive white Gaussian noise. In this work, we propose an efficient medical image denoising method that can handle a noise mixture of various types. Our method is based on block matching filtering and low-rank matrix completion as follows. A noisy slice is processed in blockwise manner and for each processed block we find similar blocks in other slices. The similar blocks then will stack together and unreliable pixels will remove using fast matrix completion method [1]. We demonstrate the effectiveness of our algorithm in removing the mixed noise through the results. Our results also proved the effectiveness of our algorithm in removing noise from regular structures. We also compare with other denoising technique using matrix completion. Our method results in comparable performance with significantly lower computation complexity.

Keywords-Medical Image denoising, Matrix Completion, Block matching

I. INTRODUCTION

Images from various modalities need to be denoised as a pre-processing step for many planning, navigation, detection, data-fusion and visualization tasks in medical applications [2], [3]. CT slices are often corrupted by noise during acquisition or transmission. Noises are added in the CT slices during acquisition by CT scanner sensors [4]. Other noise sources are introduced over transmission channels [5], [6]. Most medical image denoising algorithms proposed in the literature have assumed the additive white Gaussian noise. In contrast, we consider a noise mixture from various noise types (Impulsive/Poisson/Gaussian) in this work and will demonstrate the robustness of our medical image denoising method.

Many image denoising methods have been proposed in the last few decades, e.g., [7]–[11]. One of the first methods to address the denoising problem was the bilateral filter proposed by Tomasi and Manduchi [10]. However, this method does not perform well under strong noise. Pizurica, et al. [12] proposed a wavelet domain method for noise filtering in medical images. They exploit the general knowledge regarding the correlation of significant image features across different resolution scales to perform a preliminary coefficient classification. In [13], a wavelet-based multiscale products thresholding scheme for noise suppression of MRI was proposed. Unlike many traditional schemes that directly apply thresholding to the wavelet coefficients, their scheme first multiplies the adjacent wavelet subbands to amplify the significant features before applying thresholding to the multiscale products to better differentiate edge structures from noise.

Recently, the idea of patch based sparse coding has been applied on video denoising [7], [14]–[16]. Marial et al. in [14] attempted to improve the sparse coding approach by forcing similar patches to share the same dictionary elements in their sparse decomposition. Another notable example is block-matching 3-D filter (BM3D) [16], which utilizes an enhanced sparse representation in transform domain. In BM3D, similar 2D image blocks are grouped into 3D data array based on the l2 norm distance function. Then, the 3D data array is filtered by wavelet shrinkage or Wiener filter in 3D transform domain. The denoised image is produced from all grouped blocks after applying the inverse 3D transform. The concept of BM3D is generalized to video denoising in VBM3D [7]. In VBM3D, the noisy video is processed in a block-wise manner in both spatial and temporal domain. Then, a predictive
search block-matching is combined with collaborative hard thresholding or collaborative Wiener filtering.

However, prior works have been limited to the one specific type of noise, where existence of other types of noise will degrade the performance of the denoising methods. In contrast, our method does not suffer from this limitation and can even remove a mixture of strong noises from medical image slices.

In this work, we show that the proposed method can effectively handle noisy images that suffer from noise mixtures. The proposed method is similar to that described in [17]. However, rather than applying a suboptimal block matching algorithm described in [18], we incorporate a decomposition approach for matrix completion [1] into the denoising algorithm and use a near-optimal block matching method [19] with higher complexity. However, as the decomposition approach for matrix completion [1] has much lower computational complexity than the technique used in [17], our overall algorithm runs significantly faster than other matrix completion techniques.

The key intuition of the approach is to keep only the reliable pixels and get rid of all unreliable pixels that are likely to be overwhelmed by strong noise. For each patch in the reference slice, we find the similar patches in the other slices using block matching algorithm. The found matches will be vectorized and then stacked into a matrix. The reliable pixels in the matrix are identified based on their deviation from the mean of all elements in the same row. We will then apply matrix completion on the incomplete matrix and a nearly noise free block block will be output. Then, a denoised patch will be constructed as the average value of each row in the completed matrix. Repeating the same procedure for all blocks of reference slice will build a denoised slice.

The rest of this paper is structured as follows. In Section II, we include the implementation detail of our proposed method. We show and discuss our simulation results in Section III, followed by a brief conclusion in Section IV.

II. PROPOSED METHOD

Consider an observed noisy CT slices \( y(x) = z(x) + n(x) \), where \( z(x) \) is the original CT slices and \( n(x) \) is Gaussian/Poisson/Impulsive noise sample. \( x = (i, j, k) \in X \) are coordinates in the spatio-temporal 3D domain \( X \subset \mathbb{Z}^3 \), where the first two components \((i, j)\) are the spatial coordinates and the third one \(k\) is the time (slice) index. The main procedure for our proposed denoising method is summarized in 1.

**Algorithm 1** Medical Image Denoising using matrix completion

**Inputs**: noisy CT slices \( y \), pixel overlap \( v \)

**Initialize**:
- Set \( V \) and \( W \) to be zero images with the same size as that of an input CT slice.

**Produce the pre-processing step for removing impulsive noise before patch matching**:
- Apply Adaptive Median Filter: \( Y^{am} = AMF(y) \)

**Find the denoised patches**: For each coordinate \( x \in \Omega \) with \( v \) pixel overlap in each direction do:
  (a) \( S_x = BM\left(Y_x^{am}\right) \)
  (b) \( \hat{Z}_{S_x} = ReliableElements(Y_{S_x}) \)
  (c) \( \hat{Z}_x = DMC\left(\hat{Z}_{S_x}\right) \)
  (d) \( \hat{z}_x = AVG_{row}(\hat{Z}_x) \)
  (e) \( V = V + \hat{z}_x \)
  (f) \( W = W + \hat{w}_x \)

**Normalize**: \( \hat{Z} = V/W \)

**Output**: a denoised CT slice \( \hat{Z} \)

**Detail Explanations**:
- \( AMF(y) \) performs adaptive median filter on \( y \). Since CT slices are corrupted by noise, applying patch matching algorithm directly on noisy CT slices results in unpredictable outcome. Namely, block matching algorithm will suffer from impulsive noise and its performance will be seriously degraded from strong impulsive noise. Hence, using a preprocessing step to remove impulsive noise before block matching step will improve the result performance. In our work, we simply use the adaptive median filter proposed by Hwang et al. in [11].
- \( \Omega \subset X \) is a set that includes the coordinates of the reference blocks. In general, each pixel in the reference slice is covered by several patches, we aggregate overlapped patches by a weighted average at each pixel.
- \( Y_x^{am} \) denotes a block of size \( q \times q \) in \( Y^{am} \), where its center is at \( x \).
- \( BM\left(Y_x^{am}\right) \) denotes the block matching step taking \( Y_x^{am} \) as a reference block, where the result...
is the set $S_x$ containing the coordinates of the matched blocks. Although there are several methods to find the similar matches [7], [20], [21], in our work, we use Adaptive Rood Pattern Search (ARPS) algorithm [19] because of its computational efficiency.

- $Y_{S_x}$ denotes a matrix formed by stacking the vectorized blocks $Y_{x \in S_x}$ together, where $Y_x$ is a block of size $q \times q$ centered at $x$ in $y$.
- $\text{ReliableElements}(Y_{S_x})$ discards those matrix elements of $Y_{S_x}$, which are far away from the mean of its corresponding row and indicates them as unreliable elements, and then replaces them by zero. Note that those unreliable elements could be the pixels corrupted by Gaussian/Poisson/Impulsive noise or from mismatched patches obtained from previous step (i.e., block matching). Keeping the reliable elements lets us to recover the full matrix needed for next step.
- $\text{DMC}(\hat{Z}_{S_x})$ denotes the low-rank matrix completion step aking $\hat{Z}_{S_x}$ as input and output $\tilde{Z}_x$ as a completed matrix with removed noise elements.
- $\text{AVG}_{\text{row}}(\tilde{Z}_x)$ finds the average value of each row in matrix $\tilde{Z}_x$ and convert the obtained vector to a block. Also, $\tilde{z}_x$ will be an estimated block of size $q \times q$ centered at $x$ in $\tilde{Y}$.
- $\hat{w}_x$ is a patch with the same size as $\hat{z}_x$, which will be used to normalize $\hat{V}$. Note that, all pixel values in $\hat{w}_x$ are equal to 1.

### III. EXPERIMENTAL RESULTS

To evaluate the performance of the proposed method, we consider in two separated subsections with two different sets of experiments. In Section III-A, we will first generate synthetic noisy slices and then apply denoising methods to the degraded slices. We will then compare the results to the ground truth (the original slices) and results generated from other state-of-the-art techniques [12], [13], [25]. Moreover, in Section III-B, we will illustrate additional examples that will assess our denoising method for real CT slices. Comparison will be made against two state-of-the-art techniques: the adaptive multiscale image denoising algorithm [13] and wavelet domain image denoising algorithm [12].

![Fig. III.1: PSNR values of non-local means algorithm [25]; wavelet domain image denoising algorithm [12]; adaptive multiscale image denoising algorithm [13]; result of the proposed denoising algorithm for lung CT slices; Note that, we kept the Gaussian noise constant in all tests.](image)

We also replaced our proposed decomposition matrix completion with OptSpace [24] to compare the result and time consumption (Table II). It can be seen that our method compares well or even betters in terms of PSNR, and it performs notably faster than OptSpace [24].

### III-A. Evaluation On Synthetic Slices

To evaluate the performance of our approach, we conducted tests on the data sets LIDC-IDRI [26] where the size of each slice of the CT slices are $512 \times 512$ pixels. All tests in this section were processed in the following manner: All 6 slices were involved in the denoised slice. The similar block size used for block matching was $8 \times 8$ and was not changed for various tests. We obtained a locally consistent solution by allowing patches to overlap, where the overlapped regions ($\nu$) were 5 pixels in each direction. Further, for each reference patch, we extract 5 most similar patches used in each slice using block matching algorithm.

In Fig. III.2, we show the PSNR result and a clear visual comparison on the CT slices. The original CT slice is corrupted by a mixture of Poisson noise, Gaussian white noise, impulsive noise with significant
Table I: PSNR of our proposed denoising method for the lung slices; note that, we kept the variances of Gaussian and Poisson noises constant in all tests.

<table>
<thead>
<tr>
<th>Noise density of Impulsive noise</th>
<th>Lung Slices (362 x 362)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>24.10</td>
</tr>
<tr>
<td>0.06</td>
<td>24.05</td>
</tr>
<tr>
<td>0.11</td>
<td>24.04</td>
</tr>
<tr>
<td>0.16</td>
<td>24.04</td>
</tr>
<tr>
<td>0.21</td>
<td>24.07</td>
</tr>
<tr>
<td>0.26</td>
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</tr>
<tr>
<td>0.36</td>
<td>24.05</td>
</tr>
<tr>
<td>0.41</td>
<td>24.07</td>
</tr>
<tr>
<td>0.46</td>
<td>24.06</td>
</tr>
</tbody>
</table>

noise level (variance of Gaussian noise = 0.02 and noise density of impulsive noise = 0.01). As shown in the figure, non-local means algorithm [25], wavelet domain image denoising algorithm [12], and adaptive multiscale image denoising algorithm [13] generate severe artifacts at edge areas, while our proposed denoising method performs remarkably well for the detail structures and is free of these artifacts.

To quantify our denoising performance, we used the Peak Signal to Noise Ratio (PSNR) measure between the ground truth and denoised CT slice:

\[
PSNR = 10 \log_{10}(255^2/\text{Mean Square Error})[dB].
\] (III.1)

In Table I, we present the PSNR results of the proposed denoising algorithm for a lung CT slices; where impulsive noise is changing. This table shows how our algorithm is robust in denoising of the corrupted slices by serious impulsive noise. In graph III.1, we compare our denoising method with adaptive multiscale image denoising algorithm [13], wavelet domain image denoising algorithm [12], and non-local means algorithm [25], which are among the state-of-the-art in medical image denoising.

In this comparison, we apply our denoising method on lung slices which we changed the impulsive noise and kept the Poisson and Gaussian noise constant for all methods. Our proposed method surpasses the other methods with a significant margin for all additive Impulsive Noise.

III-B. Evaluation On Non-Synthetic Slices

In this section, we turn to some real slices, where we apply our proposed method without any changes or generating noisy slices. Note that since there are no published methods that perform denoising on such general slices, we choose the adaptive multiscale image denoising algorithm [13] and wavelet domain image denoising algorithm [12] for comparison, because their source code is available. As for the non-synthetic case, while we do not have the ground truth and thus cannot evaluate the methods quantitatively using PSNR,
Fig. III.3: Non-synthetic (real) experiment. From left to right: real CT slice; adaptive multiscale image denoising algorithm [13]; wavelet domain image denoising algorithm [12]; result of the proposed denoising algorithm.

Table II: PSNR and time comparison for using various matrix completion for lung slices

<table>
<thead>
<tr>
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<th></th>
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</thead>
<tbody>
<tr>
<td><strong>PSNR</strong></td>
<td>21.83</td>
<td>21.65</td>
</tr>
<tr>
<td><strong>Time (seconds)</strong></td>
<td>118</td>
<td>1398</td>
</tr>
</tbody>
</table>

the visual comparison illustrates the robustness of our proposed method when it is applied directly to real slices. Note that we used the same slices number, block size, overlapped region, and extracted patches number for the non-synthetic experiment just as the synthetic case.

IV. CONCLUSION

In conclusion, we have proposed in this paper an efficient patch based medical image denoising using decomposition approach for matrix completion [1], where we keep only reliable pixels and get rid of all unreliable pixels. Our method can handle a mix-
ture of noises while most of the existing methods have been limited to the one specific type of noise. Quantitative and qualitative experiments have shown that the proposed algorithm outperforms the state-of-the-art methods in handling medical image denoising.

Our future work would include developing better block matching algorithm and applying the proposed method to super resolution application.

V. REFERENCES


